Interest Rates and Return Measurement

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Interest Rates

Uses of interest rates

- As required rates of return for investments
- As discount rates to change future values to present values
- As an opportunity cost of current consumption

Risk-free rates

- A risk-free rate is a theoretical rate with no default risk
- A real risk-free rate has no expected inflation or default risk

Nominal Interest Rates

Nominal risk-free rate

- (1 + nominal RFR) = (1 + real RFR) × (1 + expected inflation)

Approximation

- Nominal risk-free rate = real risk-free rate + expected inflation

Required Returns

Securities may have one or more types of risks. Each increases the **required return** on the security:

- Default risk—a borrower may not make the promised payments
- Liquidity risk—receiving less than fair value; unable to sell
- Maturity risk—longer-dated bonds have higher interest rate risk

Risk premiums

- Each factor is associated with a risk premium required by investors
- A required return is sum of the real risk-free rate plus the relevant risk premiums

Components of Interest Rates

Required nominal interest rate on a security =

real risk-free rate
+ expected inflation
+ default risk premium
+ liquidity risk premium
+ maturity risk premium

nominal risk-free rate (real risk-free rate + expected inflation)

risk premiums (default, liquidity and maturity risk premiums)

Holding Period Returns

Investors may hold an investment for any chosen period. The HPR is the return from the start to the end of this period.

Holding period return (HPR)

- HPR no income = (end value / beginning value) – 1
- HPR with income = [(end value + dividend) / beginning value] – 1

Example:

A stock is valued at $20 at T0, and $22 at T1 and pays a $1 dividend over the period. **Calculate** the HPR.

Holding Period Return Solution

HPR = [(end value + dividend) / beginning value] – 1
= [(22 + 1) / 20] – 1
= 15%

Compounding Holding Period Returns

Holding period returns can be linked together over multiple time periods:

- HPR 3 yrs = $(1 + HPR_{yr\ 1})(1 + HPR_{yr\ 2})(1 + HPR_{yr\ 3}) – 1$

Annualized returns

- Annualized returns are commonly used rather than longer time period returns

Average Returns

The **arithmetic mean** return is the simple average of a series of periodic returns:

$$\text{Arithmetic mean} = \frac{R_1 + R_2 + R_3 ... + R_n}{N}$$

Example:

For the last three years, the returns for Acme Corporation common stock have been –9.34%, 23.45%, and 8.92%. **Calculate** the arithmetic mean return over the three-year period.

Arithmetic Mean Solution

Arithmetic mean:

$$= \frac{R_1 + R_2 + R_3 ... + R_n}{N}$$

$$= \frac{-9.34 + 23.45 + 8.92}{3}$$

= 7.68%

Geometric Mean

The geometric mean return is a compounded average return over a period.

$$GM = \sqrt[n]{(1\,,\ R_1)\cdot(1\,,\ R_2)\cdot(1\,,\ R_3)\cdot\ \ldots\ \cdot(1\,,\ R_n)} - 1$$

Example:

For the last three years, the returns for Acme Corporation common stock have been –9.34%, 23.45%, and 8.92%. **Calculate** the compound annual rate of return over the three-year period.

Geometric Mean Solution

$$GM = \sqrt[n]{(1\,,\ R_1)\cdot(1\,,\ R_2)\cdot(1\,,\ R_3)\cdot\ \ldots\ \cdot(1\,,\ R_n)} - 1$$

$$= \sqrt[3]{(1-0.0934)\cdot(1\,,\ 0.2345)\cdot(1\,,\ 0.0892)} - 1$$

$$= \sqrt[3]{1.21903} - 1$$

$$= 6.825\%$$

Arithmetic vs. Geometric Mean

The **geometric mean** will always be less than or equal to the **arithmetic mean.**

The greater dispersion of return observations, the greater the difference between the means.

The only time the **arithmetic** and **geometric means** are equal is when there is no variability in the observations.

Harmonic Mean

Harmonic mean is used to find the average cost per share of stock purchased over time, if each purchase is a constant dollar amount.

$$\overline{X}_{\text{harmonic}} = \frac{N}{\sum_{i=1}^{N} \frac{1}{X_i}}$$

N = number of purchases of equal dollar amount

X_i = share price at time *i*

Example: An investor purchases $1,000 of mutual fund shares each month. Over the last three months, the prices paid were $8, $9, and $10. **Calculate** the average cost per share.

Harmonic Mean Solution

$$\overline{X}_{harmonic} = \frac{N}{\sum_{i=1}^{N} \frac{1}{X_i}}$$

$$= \frac{3}{\frac{1}{8}, \frac{1}{9}, \frac{1}{10}}$$

= $8.926 per share

Harmonic Mean: Alternative Approach

Harmonic mean is the average cost per share of stock purchased over time.

Shares purchased

Month 1:	$1,000 / $8	= 125.00 shares
Month 2:	$1,000 / $9	= 111.11 shares
Month 3:	$1,000 / $10	= <u>100.00</u> shares
Total		= 336.11 shares

Average price paid = $3,000 / 336.11 = $8.926 per share

Harmonic, Arithmetic, and Geometric Means

What is the relationship between the means?

Arithmetic mean × harmonic mean = (geometric mean)^2

For values that are not equal

Harmonic mean < geometric mean < arithmetic mean

Appropriate uses

Geometric mean: compound returns over multiple periods

Harmonic mean: average share cost from fixed money purchases

Arithmetic mean: include all values, even outliers

Impact of Outliers

The **arithmetic mean** includes all observations, no matter how extreme. This skews the mean in the direction of an outlier.

Adjusting for outliers

The **trimmed mean** or **winsorized mean** adjusts for outliers.