2025 FRM®

Exam Prep

SchweserNotes™

Valuation and Risk Models

Part I Book 4

KAPLAN SCHWESER

Book 4: Valuation and Risk Models

SchweserNotes™ 2025

FRM Part I

KAPLAN SCHWESER

SCHWESERNOTES™ 2025 FRM® PART I BOOK 4: VALUATION AND RISK MODELS

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STUDY SESSION 12

47. Measures of Financial Risk

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 1.

After completing this reading, you should be able to:

a. describe the mean-variance framework and the efficient frontier.
b. compare the normal distribution with the typical distribution of returns of risky financial assets such as equities.
c. define the VaR measure of risk, describe assumptions about return distributions and holding periods, and explain the limitations of VaR.
d. explain and calculate ES and compare and contrast VaR and ES.
e. define the properties of a coherent risk measure and explain the meaning of each property.
f. explain why VaR is not a coherent risk measure.

48. Calculating and Applying VaR

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 2.

After completing this reading, you should be able to:

a. explain and provide examples of linear and non-linear portfolios.
b. describe and explain the historical simulation approach for computing VaR and ES.
c. describe the delta-normal approach and calculate VaR for non-linear derivatives using delta-normal approach.
d. describe and calculate VaR for linear derivatives.
e. describe the limitations of the delta-normal method.
f. explain the Monte Carlo simulation method for calculating VaR and ES and identify its strengths and weaknesses.
g. describe the implications of correlation breakdown for a VaR or ES analysis.
h. describe worst-case scenario analysis and compare it to VaR.

49. Measuring and Monitoring Volatility

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 3.

After completing this reading, you should be able to:

a. explain how asset return distributions tend to deviate from the normal distribution.
b. explain reasons for fat tails in a return distribution and describe their implications.
c. differentiate between conditional and unconditional distributions and describe regime switching.
d. compare and contrast different approaches for estimating conditional volatility.
e. apply the exponentially weighted moving average (EWMA) approach to estimate volatility, and describe alternative approaches to weighting historical return data.
f. apply the GARCH (1,1) model to estimate volatility.
g. explain and apply approaches to estimate long horizon volatility/VaR and describe the process of mean reversion according to a GARCH (1,1) model.
h. evaluate implied volatility as a predictor of future volatility and its shortcomings.
i. describe an example of updating correlation estimates.

STUDY SESSION 13

50. External and Internal Credit Ratings

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 4.

After completing this reading, you should be able to:

a. describe external rating scales, the rating process, and the link between ratings and default.

b. define conditional and unconditional default probabilities and explain the distinction between the two.
c. define hazard rate and calculate the unconditional default probability of a credit asset using hazard rate.
d. define recovery rate and calculate the expected loss from a loan.
e. explain and compare the through-the-cycle and point-in-time ratings approaches.
f. describe alternative methods to credit ratings produced by rating agencies.
g. compare external and internal ratings approaches.
h. describe, calculate, and interpret a rating transition matrix and explain its uses.
i. describe the relationships between changes in credit ratings and changes in stock prices, bond prices, and credit default swap spreads.
j. explain historical failures and potential challenges to the use of credit ratings in making investment decisions.

51. Country Risk: Determinants, Measures, and Implications

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 5.

After completing this reading, you should be able to:

a. explain how a country's economic growth rates, political risk, legal risk, and economic structure relate to its risk exposure.
b. evaluate composite measures of risk that incorporate multiple components of country risk.
c. compare instances of sovereign default in both foreign currency debt and local currency debt and explain common causes of sovereign defaults.
d. describe the consequences of sovereign default.
e. describe factors that influence the level of sovereign default risk; explain and assess how rating agencies measure sovereign default risks.
f. describe the characteristics of sovereign credit spreads and sovereign credit default swaps (CDS) and compare the use of sovereign spreads to credit ratings.

52. Measuring Credit Risk

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 6.

After completing this reading, you should be able to:

a. explain the distinctions between economic capital and regulatory capital and describe how economic capital is derived.
b. describe the degree of dependence typically observed among the loan defaults in a bank's loan portfolio, and explain the implications for the portfolio's default rate.
c. define and calculate expected loss (EL).
d. define and explain unexpected loss (UL).
e. estimate the mean and standard deviation of credit losses assuming a binomial distribution.
f. describe the Gaussian copula model and its application.
g. describe and apply the Vasicek model to estimate default rate and credit risk capital for a bank.
h. describe the CreditMetrics model and explain how it is applied in estimating economic capital.
i. describe and apply Euler's theorem to determine the contribution of a loan to the overall risk of a portfolio.
j. explain why it is more difficult to calculate credit risk capital for derivatives than for loans.
k. describe challenges to quantifying credit risk.

53. Operational Risk

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 7.

After completing this reading, you should be able to:

a. describe the different categories of operational risk and explain how each type of risk can arise.
b. compare the basic indicator approach, the standardized approach, and the advanced measurement approach for calculating operational risk regulatory capital.
c. describe the standardized measurement approach and explain the reasons for its introduction by the Basel Committee.
d. explain how a loss distribution is derived from an appropriate loss frequency distribution and loss severity distribution using Monte Carlo simulation.
e. describe the common data issues that can introduce inaccuracies and biases in the estimation of loss frequency and severity distributions.

f. describe how to use scenario analysis in instances when data are scarce.
g. describe how to identify causal relationships and how to use Risk and Control Self-Assessment (RCSA), Key Risk Indicators (KRIs), and education to understand and manage operational risks.
h. describe the allocation of operational risk capital to business units.
i. explain how to use the power law to measure operational risk.
j. explain how the moral hazard and adverse selection problems faced by insurance companies relate to insurance against operational risk.

54. Stress Testing

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 8.

After completing this reading, you should be able to:
a. describe the rationale for the use of stress testing as a risk management tool.
b. describe the relationship between stress testing and other risk measures, particularly in enterprise-wide stress testing.
c. describe stressed VaR and stressed ES, including their advantages and disadvantages, and compare the process of determining stressed VaR and ES to that of traditional VaR and ES.
d. explain key considerations and challenges related to developing stress testing scenarios and building stress testing models.
e. describe reverse stress testing and describe an example of regulatory stress testing.
f. describe the responsibilities of the board of directors, senior management, and the internal audit function in stress testing governance.
g. describe the role of policies and procedures, validation, and independent review in stress testing governance.
h. describe the Basel stress testing principles for banks regarding the implementation of stress testing.

STUDY SESSION 14

55. Pricing Conventions, Discounting, and Arbitrage

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 9.

After completing this reading, you should be able to:
a. define discount factor and calculate present and future values using a discount function.
b. define the "law of one price," explain it using an arbitrage argument, and describe how it can be applied to bond pricing.
c. identify arbitrage opportunities for fixed-income securities with certain cash flows.
d. identify the components of a U.S. Treasury coupon bond and compare the structure to Treasury STRIPS, including the difference between P-STRIPS and C-STRIPS.
e. construct a replicating portfolio using multiple fixed-income securities to match the cash flows of a given fixed-income security.
f. differentiate between "clean" and "dirty" bond pricing and explain the implications of accrued interest with respect to bond pricing.
g. describe the common day-count conventions used to calculate interest on a fixed-income security.

56. Interest Rates

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 10.

After completing this reading, you should be able to:
a. calculate and interpret the impact of different compounding frequencies on a bond's value.
b. define spot rate and calculate discount factors given spot rates.
c. interpret the forward rate and calculate forward rates given spot rates.
d. define par rate and describe how to determine the par rate of a bond.
e. interpret the relationship between spot, forward, and par rates.
f. assess the impact of a change in time to maturity on the price of a bond.
g. define the "flattening" and "steepening" of rate curves and describe a trade to reflect expectations that a curve will flatten or steepen.
h. describe a swap transaction and explain how a swap market defines par rates.

57. Bond Yields and Return Calculations

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 11.

After completing this reading, you should be able to:

a. differentiate between gross and net realized returns and calculate the realized return for a bond over a holding period including reinvestments.
b. define and interpret the spread of a bond and explain how a spread is derived from a bond price and a term structure of rates.
c. define, interpret, and apply a bond's yield to maturity (YTM) to bond pricing.
d. explain how to calculate a bond's YTM given its structure and price.
e. calculate the price of an annuity and a perpetuity.
f. explain the relationship between spot rates and YTM.
g. define the coupon effect and explain the relationship between coupon rate, YTM, and bond prices.
h. explain the decomposition of the profit and loss (P&L) for a bond position or portfolio into separate factors including carry roll-down, rate change, and spread change effects.
i. describe the common assumptions made about interest rates when calculating carry roll-down, and calculate carry roll-down under these assumptions.

58. Applying Duration, Convexity, and DV01

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 12.

After completing this reading, you should be able to:

a. describe a one-factor interest rate model and identify common examples of interest rate factors.
b. define and calculate the DV01 of a fixed-income security given a change in rates and the resulting change in price.
c. calculate the face amount of bonds required to hedge an interest rate-sensitive position given the DV01 of each.
d. define, calculate, and interpret the effective duration of a fixed-income security given a change in rates and the resulting change in price.
e. compare and contrast DV01 and effective duration as measures of price sensitivity.
f. define, calculate, and interpret the convexity of a fixed-income security given a change in rates and the resulting change in price.
g. calculate the DV01, duration, and convexity of a portfolio of fixed-income securities.
h. explain the hedging of a position based on effective duration and convexity.
i. construct a barbell portfolio to match the cost and duration of a given bullet investment and explain the advantages and disadvantages of bullet and barbell portfolios.

59. Modeling Non-Parallel Term Structure Shifts and Hedging

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 13.

After completing this reading, you should be able to:

a. describe principal components analysis and identify the factors that are the most important drivers of term structure movements.
b. describe key rate shift analysis and define key rate 01 (KR01).
c. calculate the KR01s of a portfolio given a set of key rates.
d. calculate the positions in hedging instruments necessary to hedge the key rate risks of a portfolio.
e. apply key rate analysis and principal components analysis to estimating portfolio volatility.
f. describe an interest rate bucketing approach, define forward bucket 01, and compare forward bucket 01s to KR01s.
g. calculate the corresponding duration measure given a KR01 or forward bucket 01.

STUDY SESSION 15

60. Binomial Trees

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 14.

After completing this reading, you should be able to:

a. calculate the value of an American and a European call or put option using a one-step and two-step binomial model.

b. describe how volatility is captured in the binomial model.
c. describe how the value calculated using a binomial model converges as time periods are added.
d. define and calculate delta of a stock option.
e. explain how the binomial model can be altered to price options on stocks with dividends, stock indices, currencies, and futures.

61. The Black-Scholes-Merton Model

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 15.

After completing this reading, you should be able to:

a. explain the lognormal property of stock prices, the distribution of rates of return, and the calculation of expected return.
b. calculate the realized return and historical volatility of a stock.
c. describe the assumptions underlying the Black-Scholes-Merton option pricing model.
d. calculate the value of a European option on a non-dividend-paying stock using the Black-Scholes-Merton model.
e. define implied volatilities and describe how to calculate implied volatilities from market prices of options using the Black-Scholes-Merton model.
f. explain how dividends affect the decision to exercise early for American call and put options.
g. calculate the value of a European option on a dividend-paying stock, futures, or foreign currency using the Black-Scholes-Merton model.
h. describe warrants, calculate the value of a warrant, and calculate the dilution cost of the warrant to existing shareholders.

62. Option Sensitivity Measures: The "Greeks"

Global Association of Risk Professionals. *Valuation and Risk Models*. New York, NY: Pearson, 2022. Chapter 16.

After completing this reading, you should be able to:

a. describe and assess the risks associated with naked and covered option positions.
b. describe the use of a stop-loss hedging strategy, including its advantages and disadvantages, and explain how this strategy can generate naked and covered option positions.
c. calculate the delta of an option.
d. explain delta hedging for an option position, including its dynamic aspects.
e. define and describe vega, gamma, theta, and rho for option positions and calculate the gamma and vega of an option.
f. explain how to implement and maintain a delta-neutral and gamma-neutral position.
g. describe the relationship between delta, theta, gamma, and vega.
h. calculate the delta, gamma, and vega of a portfolio.
i. describe how to implement portfolio insurance and how this strategy compares with delta hedging.

The following is a review of the Valuation and Risk Models principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Valuation and Risk Models, Chapter 1.

READING 47

MEASURES OF FINANCIAL RISK

Study Session 12

EXAM FOCUS

The assumption regarding the shape of the underlying return distribution is critical in determining an appropriate risk measure. The mean-variance framework can only be applied under the assumption of an elliptical distribution, such as the normal distribution. The value at risk (VaR) measure can calculate risk measures when the return distribution is nonelliptical, but the measurement is unreliable and no estimate of the amount of loss is provided. Expected shortfall is a more robust risk measure that satisfies all the properties of a coherent risk measure with less restrictive assumptions. For the exam, focus your attention on the calculation of VaR, properties of coherent risk measures, and the expected shortfall methodology.

MODULE 47.1: PORTFOLIO THEORY AND VALUE AT RISK

Mean-Variance Framework

LO 47.a: Describe the mean-variance framework and the efficient frontier.

The traditional mean-variance model estimates the amount of financial risk for portfolios in terms of the portfolio's expected return (i.e., mean) and risk (i.e., standard deviation or variance). Under the **mean-variance framework**, it is necessary to assume that return distributions for portfolios are elliptical distributions. The most commonly known elliptical probability distribution function is the normal distribution.

The **normal distribution** is a continuous distribution that illustrates all possible outcomes for random variables. Recall that the standard normal distribution has a mean of zero and a standard deviation of one. If returns are normally distributed, approximately 66.7% of returns will occur within plus or minus one standard deviation of the mean, and approximately 95% of the observations will occur within plus or minus two standard deviations of the mean. Thus, given this type of distribution, returns are more likely to occur closer to the mean return.

For a portfolio, the following equations are used to calculate the mean and standard deviation. A two-asset portfolio will have the following equations, with *w* reflecting weights in a portfolio, μ equaling expected returns, σ equaling standard deviations, and ρ representing the correlation coefficient.

$$\text{Mean: } \mu_P = w_1\mu_1 + w_2\mu_2$$

$$\text{Standard deviation: } \sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho}$$

For a portfolio with *n* investments, the mean and standard deviation of the portfolio are as follows:

$$\text{Portfolio mean: } \mu_P = \sum_{i=1}^{n} w_i\mu_i$$

$$\text{Portfolio standard deviation: } \sigma_P = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_iw_j\sigma_i\sigma_j\rho_{ij}}$$

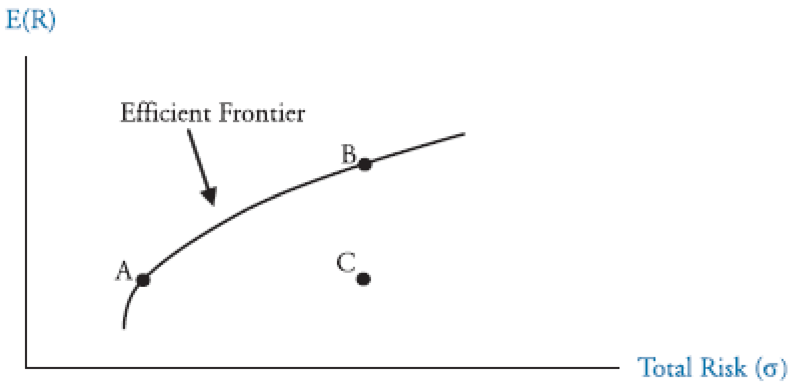
Portfolio managers are concerned with measuring downside risk and, therefore, are particularly interested in measuring the possibility of outcomes to the left or below the expected return. If the return distribution is symmetrical (like the normal distribution), then the standard deviation is an appropriate measure of risk when determining the probability that an undesirable outcome will occur.

If we assume that return distributions for all risky securities are normally distributed, then we can choose portfolios based on the expected returns and standard deviations of all possible combinations of risky securities. Figure 47.1 illustrates the concept of the **efficient frontier**.

In theory, all investors prefer securities or portfolios that lie on the efficient frontier. Consider Portfolios A, B, and C in Figure 47.1. If you had to choose between Portfolios A and C, which one would you prefer and why? Since Portfolios A and C have the same expected return, a risk-averse investor would choose the portfolio with the least amount of risk (which would be Portfolio A). Now if you had to choose between Portfolios B and C, which one would you choose and why? Because Portfolios B and C have the same amount of risk, a risk-averse investor would choose the portfolio with the higher expected return (which would be Portfolio B). We say that Portfolio B dominates Portfolio C with respect to expected return, and that Portfolio A dominates Portfolio C with respect to risk. Likewise, all portfolios on the efficient frontier dominate all other portfolios in the investment universe of risky assets with respect to either risk, return, or both.

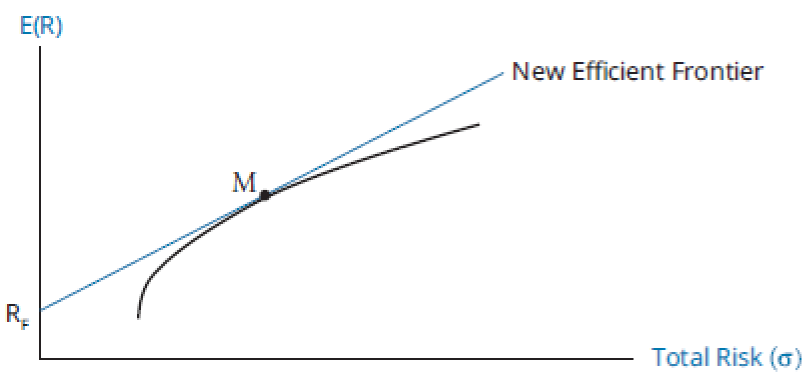
There are an almost unlimited number of combinations of risky assets to the right and below the efficient frontier. However, in the absence of a risk-free security, portfolios to the left and above the efficient frontier are not possible. Therefore, all investors will choose some portfolio on the efficient frontier. If an investor is more risk-averse, she may choose a portfolio on the efficient frontier closer to Portfolio A. If an investor is less risk-averse, she will choose a portfolio on the efficient frontier closer to Portfolio B.

Figure 47.1: The Efficient Frontier



If we now assume that there is a risk-free security, then the mean-variance framework is extended beyond the efficient frontier. Figure 47.2 illustrates that the optimal set of portfolios now lie on a straight line (the new efficient frontier) that runs from the risk-free security through the **market portfolio**, *M*. The market portfolio consists (in theory) of all investments available in the market in their respective proportions. All investors will now seek investments by holding some portion of the risk-free security and the market portfolio. To achieve points on the line to the right of the market portfolio, an investor who is very aggressive will borrow money (at the risk-free rate) and invest in more of the market portfolio. More risk-averse investors will hold some combination of the risk-free security and the market portfolio to achieve portfolios on the line segment between the risk-free security and the market portfolio.

Figure 47.2: The Efficient Frontier With the Risk-Free Security



Mean-Variance Framework Limitations

LO 47.b: Compare the normal distribution with the typical distribution of returns of risky financial assets such as equities.

In order to apply the mean-variance framework, certain assumptions must be made. First, the means, standard deviations, and correlations between investment returns are assumed to be consistent from the perspective of all investors. Second, there is an assumption that the mean and the standard deviation are all that matters in regard to portfolios. Finally, all investors are assumed to be able to borrow at the risk-free rate of interest. Each of these assumptions are challenging from a practical perspective.

Another limitation is that the use of the standard deviation as a risk measure is not appropriate for nonnormal distributions. If the shape of the underlying return density function is not symmetrical, then the standard deviation does not capture the appropriate probability of obtaining undesirable return outcomes.

As noted earlier, the standard normal distribution has a mean of zero and a standard deviation of one. The cumulative distribution for the standard normal distribution is estimated by calculating a *z*-score. The cumulative probability of the area under the standard normal distribution up to a specific value (x) can be found using the following formula:

$$z = \frac{x - \mu}{\sigma}$$

For example, if the mean is three and the standard deviation is six, to determine the cumulative probability that the value is less than two, the *z*-score is calculated as:

$$z = \frac{2 - 3}{6} = -0.1667$$

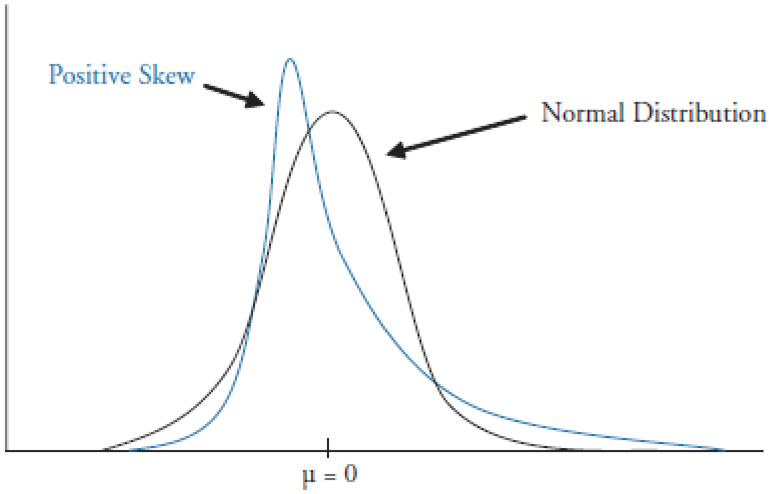
A *z*-score of –0.1667 equates to a probability of 0.4338, or 43.4%. This means that the cumulative probability that the value will be two or below is equal to 43.4%.

The assumption with financial variables is that they follow a normal distribution. However, for risky financial assets like equities, the distribution tends to have fatter tails and be more peaked than the normal distribution. This implies that extreme events are more likely in reality than what is predicted based on the normal distribution.

For example, in reviewing the returns of the S&P 500 over the 20-year period from 1998–2017, actual returns and standard deviations showed that small and large changes happen much more frequently than would be expected if returns followed a normal distribution.

Figure 47.3 illustrates two probability distribution functions. One probability distribution function is the normal distribution with a mean of zero. The other probability distribution is positively skewed. This positively skewed distribution has the same mean and standard deviation as the normal distribution. The degree of skewness alters the entire distribution. For the positively skewed distribution, outcomes below the mean are more likely to occur closer to the mean. Clearly normality is an important assumption when using the mean-variance framework. Thus, the mean-variance framework is unreliable when the assumption of normality is not met.

Figure 47.3: Normal Distribution vs. Positively Skewed Distribution



Value at Risk

LO 47.c: Define the VaR measure of risk, describe assumptions about return distributions and holding periods, and explain the limitations of VaR.

Value at risk (**VaR**) is interpreted as the worst possible loss under normal conditions over a specified period. Another way to define VaR is as an estimate of the maximum loss that can occur with a given confidence level. If an analyst says, "for a given month, the VaR is $1 million at a 95% level of confidence," then this translates to mean "under normal conditions, in 95% of the months (19 out of 20 months), we expect the fund to either earn a profit or lose no more than $1 million." Analysts may also use other standard confidence levels (e.g., 90% and 99%). **Delta-normal VaR** can be computed using the following expression: $[\mu - (z)(\sigma)] \times$ asset value.

EXAMPLE: **Calculating value at risk**

For a $100,000,000 portfolio, the expected 1-week portfolio return and standard deviation are 0.00188 and 0.0125, respectively. **Calculate** the 1-week VaR with a 95% confidence level.

Answer:

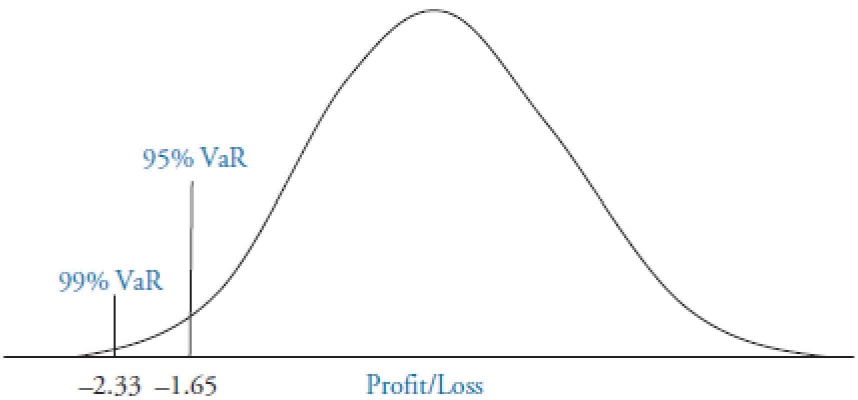
$$\begin{aligned} \text{VaR} &= (\mu - z\sigma) \times \text{portfolio value} \\ &= [0.00188 - 1.65(0.0125)] \times \$100{,}000{,}000 \\ &= -0.018745 \times \$100{,}000{,}000 \\ &= -\$1{,}874{,}500 \end{aligned}$$

The manager can be 95% confident that the maximum 1-week loss will not exceed $1,874,500.

A major limitation of the VaR measure for risk is that two arbitrary parameters are used in the calculation—the confidence level and the holding period. The confidence level indicates the likelihood or probability that we will obtain a value greater than or equal to VaR. The holding period can be any predetermined time period measured in days, weeks, months, or years.

Figure 47.4 illustrates VaR parameters for a confidence level of 95% and 99%. As you can see, the level of risk is dependent on the degree of confidence chosen. VaR increases when the confidence level increases. In addition, VaR will increase at an increasing rate as the confidence level increases.

Figure 47.4: VaR Measurements for a Normal Distribution



The second arbitrary parameter is the holding period. VaR will increase with increases in the holding period. The rate at which VaR increases is determined in part by the mean of the distribution. If the return distribution has a mean, μ, equal to 0, then VaR rises with the square root of the holding period (i.e., the square root of time). If the return distribution has a $\mu > 0$, then VaR rises at a lower rate and will eventually decrease. Thus, the mean of the distribution is an important determinant for estimating how VaR changes with changes in the holding period.

VaR estimates are also subject to both model risk and implementation risk. Model risk is the risk of errors resulting from incorrect assumptions used in the model. Implementation risk is the risk of errors resulting from the implementation of the model.

Another major limitation of the VaR measure is that it does not tell the investor the amount or magnitude of the actual loss. VaR only provides the maximum value we can lose for a given confidence level. Two different return distributions may have the same VaR, but very different risk exposures. A practical example of how this can be a serious problem is when a portfolio manager is selling out-of-the-money options. For a majority of the time, the options will have a positive return and, therefore, the expected return is positive. However, in the unfavorable event that the options expire in the money, the resulting loss can be very large. Thus, different strategies focusing on lowering VaR can be very misleading since the magnitude of the loss is not calculated.

To summarize, VaR measurements work well with elliptical return distributions, such as the normal distribution. VaR is also able to calculate the risk for nonnormal distributions; however, VaR estimates may be unreliable in this case. Limitations in implementing the VaR model for determining risk result from the underlying return distribution, arbitrary confidence level, arbitrary holding period, and the inability to calculate the magnitude of losses. The measure of VaR also violates the coherent risk measure property of subadditivity when the return distribution is not elliptical. This property is further explained later in this reading.

MODULE QUIZ 47.1

1. The mean-variance framework is inappropriate for measuring risk when the underlying return distribution:
   A. is normal.
   B. is elliptical.
   C. has a kurtosis equal to three.
   D. is positively skewed.

2. Assume an investor is very risk-averse and is creating a portfolio based on the mean-variance model and the risk-free asset. The investor will most likely choose an investment on the:
   A. left-hand side of the efficient frontier.
   B. right-hand side of the efficient frontier.
   C. line segment connecting the risk-free rate to the market portfolio.
   D. line segment extending to the right of the market portfolio.

3. An investor has purchased an equity security which is part of the S&P 500 index. Relative to the normal distribution, she can reasonably expect the security's returns to:
   A. be less peaked.
   B. exhibit zero skewness.
   C. have no excess kurtosis.
   D. be more extreme in both directions.

MODULE 47.2: COHERENT RISK MEASURES AND EXPECTED SHORTFALL

Coherent Risk Measures

LO 47.e: Define the properties of a coherent risk measure and explain the meaning of each property.

In order to properly measure risk, one must first clearly define what is meant by a measure of risk. If we allow R to be a set of random events and $\rho(R)$ to be the risk measure for the random events, then **coherent risk measures** should exhibit the following properties:

1. **Monotonicity:** a portfolio with greater future returns will likely have less risk: $R_1 \geq R_2$, then $\rho(R_1) \leq \rho(R_2)$
2. **Subadditivity:** the risk of a portfolio is at most equal to the risk of the assets within the portfolio: $\rho(R_1 + R_2) \leq \rho(R_1) + \rho(R_2)$
3. **Positive homogeneity:** the size of a portfolio, β, will impact the size of its risk: for all $\beta > 0$, $\rho(\beta R) = \beta\rho(R)$
4. **Translation invariance:** the risk of a portfolio is dependent on the assets within the portfolio: for all constants c (representing cash), $\rho(c + R) = \rho(R) - c$

The first, third, and fourth properties are more straightforward properties of well-behaved distributions. Monotonicity infers that if a random future value R_1 is always greater than a random future value R_2, then the risk of the return distribution for R_1 is less than the risk of the return distribution for R_2. Positive homogeneity suggests that the risk of a position is proportional to its size. Positive homogeneity should hold as long as the security is in a liquid market. Translation invariance implies that the addition of a sure amount reduces the risk at the same rate as the cash needed to make the position acceptable.

Subadditivity is the most important property for a coherent risk measure. The property of subadditivity states that a portfolio made up of subportfolios will have equal or less risk than the sum of the risks of each individual subportfolio. This assumes that when individual risks are combined, there may be some diversification benefits or, in the worst case, no diversification benefits and no greater risk. This implies grouping or adding risks does not increase the overall aggregate risk amount.

Expected Shortfall

LO 47.d: Explain and calculate ES and compare and contrast VaR and ES.

LO 47.f: Explain why VaR is not a coherent risk measure.

Value at risk is the minimum percent loss, equal to a pre-specified worst-case quantile return (typically the 5th percentile return). **Expected shortfall (ES)** is the expected loss given that the portfolio return already lies below the pre-specified worst-case quantile return (i.e., below the 5th percentile return). In other words, expected shortfall is the mean percent loss among the returns falling below the q-quantile. Expected shortfall is also known as **conditional VaR** or **expected tail loss (ETL)**, and by definition, must exceed VaR.

For example, assume an investor is interested in knowing the 5% VaR (the 5% VaR is equivalent to the 5th percentile return) for a fund. Further, assume the 5th percentile return for the fund equals –20%. Therefore, 5% of the time, the fund earns a return less than –20%. The value at risk is –20%. However, VaR does not provide good information regarding the expected size of the loss if the fund performs in the lower 5% of the possible outcomes. That question is answered by the expected shortfall amount, which is the expected value of all returns falling below the fifth percentile return (i.e., below –20%). Therefore, expected shortfall will equal a larger loss than the VaR.

For a normal distribution with a mean equal to μ and a standard deviation equal to σ, the following equation can be used to calculate the expected shortfall:

$$\text{ES} = \mu + \sigma \frac{e^{-(z^2/2)}}{(1 - x)\sqrt{2\pi}}$$

In this equation, x equals the confidence level and z equals the point in the distribution that has a probability of being exceeded of x%.

EXAMPLE: Calculating expected shortfall

For a $100,000,000 portfolio the expected 1-week portfolio return is zero and the standard deviation is 0.0125. **Calculate** the 1-week expected shortfall in dollar terms with a 95% confidence level.

Answer:

$$\begin{aligned}
\%ES &= \mu + \sigma \frac{e^{-(z^2/2)}}{(1 - x)\sqrt{2\pi}} \\
&= 0.0125 \frac{e^{-(1.65^2/2)}}{(1 - 0.95)\sqrt{2 \times 3.1416}} \\
&= 0.0125 \frac{0.2563}{0.12533} = 0.02556 \\
\$ES &= 0.02556 \times \$100{,}000{,}000 = \$2{,}556{,}000
\end{aligned}$$

Unlike VaR, ES has the ability to satisfy the property of subadditivity. Under this property, the risk of a portfolio should never exceed the combined risk of the assets within the portfolio. Assuming most assets have less than perfect positive correlation with each other, overall risk will decline when assets within a portfolio are combined. With VaR, the combined VaR may exceed the summation of the individual assets' VaRs; with expected shortfall, this is not the case.

The ES method provides an estimate of how large of a loss is expected if an unfavorable event occurs. VaR does not provide any estimate of the magnitude of losses, only the probability that they might occur. The property of subadditivity under the ES framework is also beneficial in eliminating another problem for VaR. When adjusting both the holding period and confidence level at the same time, an ES surface curve showing the interactions of both adjustments is convex. This implies that the ES method is more appropriate than the VaR method in solving portfolio optimization problems.

ES is similar to VaR in that both provide a common consistent risk measure across different positions. ES can be implemented in determining the probability of losses the same way that VaR is implemented as a risk measure, and they both appropriately account for correlations.

However, ES is a more appropriate risk measure than VaR for the following reasons:

- ES satisfies all of the properties of coherent risk measurements including subadditivity. VaR only satisfies these properties for normal distributions.
- The portfolio risk surface for ES is convex because the property of subadditivity is met. Thus, ES is more appropriate for solving portfolio optimization problems than the VaR method.
- ES gives an estimate of the magnitude of a loss for unfavorable events. VaR provides no estimate of how large a loss may be.
- ES has less restrictive assumptions regarding risk/return decision rules.

MODULE QUIZ 47.2

1. $\rho(X + Y) \leq \rho(X) + \rho(Y)$ is the mathematical equation for which property of a coherent risk measure?
   A. Monotonicity.
   B. Subadditivity.
   C. Positive homogeneity.
   D. Translation invariance.

2. Which of the following is not a reason that expected shortfall (ES) is a more appropriate risk measure than value at risk (VaR)?
   A. For normal distributions, only ES satisfies all the properties of coherent risk measurements.
   B. For nonelliptical distributions, the portfolio risk surface formed by holding period and confidence level is more convex for ES.
   C. ES gives an estimate of the magnitude of a loss.
   D. ES has less restrictive assumptions regarding risk/return decision rules than VaR.

KEY CONCEPTS

LO 47.a

The traditional mean-variance model estimates the amount of financial risk for portfolios in terms of the portfolio's expected return (mean) and risk (standard deviation or variance). A necessary assumption for this model is that return distributions for the portfolios are elliptical distributions.

The efficient frontier is the set of portfolios that dominate all other portfolios in the investment universe of risky assets with respect to risk and return. When a risk-free security is introduced, the optimal set of portfolios consists of a line from the risk-free security that is tangent to the efficient frontier at the market portfolio.

Assumptions needed in order to apply the mean-variance framework are (1) all investors assume the same means, standard deviations, and correlation coefficients for investments, (2) mean and standard deviation are all that matter, and (3) all investors can borrow at the risk-free rate. The mean-variance framework is unreliable when the underlying return distribution is not normal or elliptical. The standard deviation is not an accurate measure of risk and does not capture the probability of obtaining undesirable return outcomes when the underlying return density function is not symmetrical.

LO 47.b

The standard normal distribution has a mean of zero and a standard deviation of one. The z-score can be used to determine the cumulative probability of a return landing at or below a specific value. Returns on risky financial assets tend to exhibit a more peaked distribution with fatter tails, implying that the probability of more extreme events is higher than would be expected with the normal distribution.

LO 47.c

Value at risk (VaR) is a risk measurement that determines the probability of an occurrence in the left-hand tail of a return distribution at a given confidence level. VaR can be defined as $[\mu - (z)(\sigma)]$. The underlying return distribution, arbitrary choice of confidence levels and holding periods, and the inability to calculate the magnitude of losses result in limitations in implementing the VaR model when determining risk.

LO 47.d

Expected shortfall (ES) is a more accurate risk measure than VaR for the following reasons:

- ES satisfies all the properties of coherent risk measurements including subadditivity.
- The portfolio risk surface for ES is convex since the property of subadditivity is met. Thus, ES is more appropriate for solving portfolio optimization problems than the VaR method.
- ES gives an estimate of the magnitude of a loss for unfavorable events. VaR provides no estimate of how large a loss may be.
- ES has less restrictive assumptions regarding risk/return decision rules.

LO 47.e

The properties of a coherent risk measure are the following:

- Monotonicity: $Y \geq X \Rightarrow \rho(Y) \leq \rho(X)$; greater future returns imply less risk.
- Subadditivity: $\rho(X + Y) \leq \rho(X) + \rho(Y)$; portfolio risk ≤ sum of individual asset risks.
- Positive homogeneity: $\rho(\beta X) = \beta\rho(X)$ for $h > 0$; portfolio size impacts risk.
- Translation invariance: $\rho(c + X) = \rho(X) - c$; adding cash reduces risk by the amount of cash.

LO 47.f

Subadditivity, the most important property for a coherent risk measure, states that a portfolio made up of subportfolios will have equal or less risk than the sum of the risks of each individual subportfolio. VaR violates the property of subadditivity.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 47.1

1. **D** The mean-variance framework is only appropriate when the underlying distribution is elliptical. The normal distribution is a special case of elliptical distributions where skewness is equal to zero and kurtosis is equal to three. If there is any skewness, the distribution function will not be symmetrical, and standard deviation will not be an appropriate risk measure. (LO 47.a)

2. **C** Under the mean-variance framework, when a risk-free security is included in the analysis, the optimal set of portfolios lies on a straight line that runs from the risk-free security to the market portfolio. All investors will hold some portion of the risk-free security and the market portfolio. More risk-averse investors will hold some combination of the risk-free security and the market portfolio to achieve portfolios on the line segment between the risk-free security and the market portfolio. (LO 47.a)

3. **D** Returns on risky assets such as equities do not tend to follow the normal distribution. They tend to follow a distribution that is more peaked, has fatter tails, and will likely not align with the normal distribution's properties of zero skewness and zero excess kurtosis. The fatter tails indicate that more extreme returns (in both directions) are likely. (LO 47.b)

Module Quiz 47.2

1. **B** The property of subadditivity states that a portfolio made up of subportfolios will have equal or less risk than the sum of the risks of each individual subportfolio. (LO 47.e)
2. **A** VaR and ES both satisfy all the properties of coherent risk measures for normal distributions. However, only ES satisfies all the properties of coherent risk measures when the assumption of normality is not met. (LO 47.f)

The following is a review of the Valuation and Risk Models principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Valuation and Risk Models, Chapter 2.

READING 48

CALCULATING AND APPLYING VaR

Study Session 12

EXAM FOCUS

In this reading, risk measurement approaches are discussed for linear and nonlinear derivatives. Methods for calculating value at risk (VaR) and expected shortfall (ES) under the historical simulation approach, the delta-normal approach, and full revaluation approach are then discussed, including the advantages and disadvantages and underlying assumptions of the various approaches. Finally, structured Monte Carlo, stress testing, and worst-case scenario (WSC) analysis are presented as useful methods in extending VaR techniques to more appropriately measure risk for complex derivatives and scenarios.

MODULE 48.1: LINEAR AND NON-LINEAR DERIVATIVES

LO 48.a: Explain and provide examples of linear and non-linear portfolios.

A **linear derivative** reflects a relationship between an underlying factor and the derivative that is linear in nature. For example, an equity index futures contract is a linear derivative, while an option on the same index is nonlinear. The delta (rate of change) for a linear derivative must be constant for all levels of the underlying factor, but not necessarily equal to one.

A forward contract on an asset is a linear derivative because the forward contract is a linear function of the asset. The value of the forward contract at any time prior to expiry can be expressed as:

forward value = S – PV(K)

where S is the current asset price and $PV(K)$ is the present value of the asset price at a future time T. This reflects a linear relationship.

The value of a **nonlinear derivative** is a function of the change in the value of the underlying asset and is dependent on the state of the underlying asset, and reflects a relationship between an underlying factor and the derivative that is not linear in nature. A call option is a good example of a nonlinear derivative. At expiry, the call option payoff is zero if the asset price is below the strike price, but if the asset price is above